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SELF EXCITED OSCILLATIONS IN JOURNAL BEARINGS

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Under certain rotation rates, a rotor, supported by an oil-lubricated journal bearing, can experience excessive shaft vibration. Two types of oscillations can exist called oil whirl and oil whip. Oil whirl produces a circular orbit in a direction of the same sense as the shaft rotation but at a value of about 45% of the shaft rotation speed. Oil whip is a more violent oscillation which occurs at the resonance frequency of the shaft system. Both of these oscillation are predicted in this report by obtaining an analytic

description of the fluid forces and inserting these into the dynamic equations for the rotor system. Oil whirl is shown to be an auto rotation, Ω, of the journal centroid at a rate of one half of the shaft rotational speed. Oil whip is shown to occur when the frequency of autorotation exceeds the natural frequency of the shaft system, i.e. $\Omega \geq \sqrt{k/m}$.

FOREWORD

This report is the result of research performed in the Computational Aerodynamics Group, Aerodynamics and Airframe Branch, Aeromechanics Division, Flight Dynamics Laboratory from June 1981 to September 1981. This report was prepared under Work Unit 2307N603 "Computational Fluid Dynamics". Dr Wilbur L. Hankey is the Task Manager.

The author is indebted to Dr Agnieszka Muszynska for suggesting this research topic and for fruitful discussions concerning the physical phenomena.

The material presented herein is related to several other efforts on self-excited oscillations being accomplished in the Computational Aerodynamics Group.

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LIST OF SYMBOLS

a	radius of inner cylinder
b	difference in radii between inner and outer cylinders
c	damping coefficient
C_q	rotary derivative of force coefficient
e	eccentricity of cylinder
F_r, F_θ	polar coordinate components of fluid force
g	acceleration of gravity
h	gap thickness = $b(1-e\cos\theta')$
k	bending spring constant of shaft
L	width of journal bearing
m	mass of rotor system
n	mass unbalance moment arm
p	pressure
q	rotational rate perturbation = $\dot{\theta}-\Omega$
Q	volume flow rate between cylinders
r,θ	polar coordinates
t	time
u	fluid velocity in θ direction
v	fluid velocity in r direction
x,y	Cartesian coordinates
X,Y	Cartesian components of fluid force
z	gap coordinate = $r-a$
α	constant = $\dfrac{Q}{ab(\frac{\omega}{2}-\dot{\theta})}$
β	coefficient = $2(1-\alpha-e^2)$
δ	radial perturbation of orbit

LIST OF SYMBOLS (Cont'd)

μ	fluid dynamic viscosity
σ	parameter $= \dfrac{(2+e^2)}{6e^2}\left(1 - \sqrt{1-e^2}\right)$
τ	shear stress
ω	shaft forced rotation rate
Ω	autorotation rate
λ	eigenvalue of characteristic equation

SECTION I

INTRODUCTION

Under certain rotation rates, a rotor, supported by an oil-lubricated journal bearing, can experience excessive shaft vibration (Fig. 1). Two types of oscillations can exist called oil whirl and oil whip (Ref 1-6). Oil whirl produces a circular orbit in a direction of the same sense as the shaft rotation but at a value of about 45% of the shaft rotational speed (Fig. 2). Oil whip is a more violent oscillation which occurs at the resonance frequency of the shaft system. Although this phenomenon has been investigated for some time and identified as a self-excited oscillation due to viscous effects, no satisfactory analytic prediction method has been developed. It is the purpose of this note to provide such an analysis.

SECTION II

GOVERNING DYNAMIC EQUATIONS

Consider an unbalanced rotor with mass, m, shaft elasticity, k, rotating at speed, ω, under the influence of fluid forces, X and Y (Ref. 1, 2).

Hence

$$m\ddot{x} + kx = m n \omega^2 \cos \omega t + X$$

$$m\ddot{y} + ky = m n \omega^2 \sin \omega t - mg + Y$$

The classical method (Ref. 2) for describing the damping forces is

$$X = -c\dot{x}$$

$$Y = -c\dot{y}$$

The two dynamic equations are uncoupled and can be readily solved as follows (for small damping, in which $(\frac{c}{2m})^2 << \frac{k}{m}$):

$$x = r_o e^{-\frac{ct}{2m}} \cos (\omega_o t + \phi) + \frac{n\omega^2}{\omega_o^2 - \omega^2} \cos \omega t$$

$$y = r_o e^{-\frac{ct}{2m}} \sin (\omega_o t + \phi) + \frac{n\omega^2}{\omega_o^2 - \omega^2} \sin \omega t - \frac{g}{\omega_o^2}$$

where

$\omega_o^2 = \frac{k}{m}$; r_o and ϕ are ascertained from initial conditions.

These results, however, give little insight into the cause of oil whirl or whip. The reason for the failure is believed to be the inability to adequately describe the fluid forces. Therefore another mathematical model for the prediction of these forces was developed.

SECTION III

GOVERNING FLUID MECHANIC EQUATIONS

In this section the fluid motion between two non-concentric rotating cylinders will be analyzed to obtain the fluid dynamic force on the inner cylinder representative of a journal bearing (Fig. 3). The radius of the inner cylinder is 'a' and the radius of the outer cylinder is 'a+b', where $\frac{b}{a} << 1$. To analyze this problem two coordinate systems must be defined, i.e., an inertial system which is fixed at the center of the outer cylinder (x,y) and a rotating coordinate system with a moving origin referenced to the center of the inner cylinder (x', y'). The set of polar coordinates associated with these two systems are (r,θ) and (r',θ') respectively. The inner cylinder is considered to be rotating at a constant value of $\dot{\theta}$ about the center of the outer cylinder with an orbit radius of eb. The incompressible Navier-Stokes equations in polar coordinates for this rotating system are as follows:

$$\frac{\partial u}{\partial \theta'} + \frac{\partial}{\partial r'}(r'v) = 0$$

$$\frac{Du}{Dt} + \frac{uv}{r'} = -\frac{\partial p}{\rho r' \partial \theta'} + \nu\left(\nabla^2 u - \frac{u}{r'^2} - \frac{2\partial v}{r'^2 \partial \theta'}\right) - 2\dot{\theta}v$$

$$\frac{Dv}{Dt} - \frac{u^2}{r'} = -\frac{\partial p}{\rho \partial r'} + \nu\left(\nabla^2 v - \frac{v}{r'^2} - \frac{2}{r'^2 \partial \theta'}\right) + \dot{\theta}(2u + r'\dot{\theta})$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial r'} + u\frac{\partial}{r'\partial \theta'}$$

$$\nabla^2 = \frac{\partial^2}{\partial r'^2} + \frac{1}{r'}\frac{\partial}{\partial r'} + \frac{\partial^2}{r'^2 \partial \theta'^2}$$

Consistent with lubrication theory these equations may be greatly simplified for small gap thicknesses.

Let $r' = a+z$

where $\frac{z}{a} << 1$

and $\frac{v}{u} << 1$

where z has limits between zero and h, the gap thickness. Using the law of cosines a relationship for h may be obtained.

$$(a+b)^2 = (be)^2 + (a+h)^2 - 2be\,(a+h)\cos(\pi-\theta')$$

For the case at hand, $\frac{b}{a} << 1$, hence

$h(\theta')=b\,(1-e\cos\theta')$ is a good approximation.

An order of magnitude analysis of the Navier-Stokes equations produces the classical lubrication equations.

$$Q = \int_0^h u\,dz = \text{flow rate}$$

$$\frac{dp}{ad\theta'} = \mu\,\frac{\partial^2 u}{\partial z^2} = \text{pressure gradient}$$

with boundary conditions

$$u(\theta',\, z=0) = (\omega-\dot{\theta})\,a$$

$$u(\theta',\, z=h) = -\,\dot{\theta}a$$

The pressure must satisfy a periodic boundary condition.

$$p(\theta') = p(\theta'+2\pi)$$

Integrating the pressure gradient equation twice with respect to z produces a relationship for u.

$$u = \frac{dp}{a\mu d\theta'}\,\frac{z^2}{2} + C_1 z + C_2$$

Applying the two boundary conditions for u on the cylinder walls eliminates the two integration constants.

$$u+\dot{\theta}a = \frac{dp}{2a\mu d\theta'}\,z(z-h) - \frac{\omega a}{h}\,(z-h)$$

Note that from this expression the shear stress on the wall of the inner cylinder can be obtained as follows:

$$\tau=\mu \frac{\partial u}{\partial z} (z=0) = - \frac{h}{2a\mu} \frac{dp}{d\theta'} - \frac{\mu a \omega}{h}$$

Integrating u with respect to z produces the flow rate.

$$Q = \int_o^h udz = (\frac{\omega}{2} - \dot{\theta}) ah - \frac{h^3}{12a\mu} \frac{dp}{d\theta'}$$

or rearranging this equation.

$$\frac{b^2 dp/d\theta'}{12a^2\mu(\frac{\omega}{2} - \dot{\theta})} = \frac{1-\alpha-e \cos\theta'}{(1-e \cos \theta')^3}$$

where

$$\alpha = \frac{Q}{ab(\frac{\omega}{2} - \dot{\theta})}$$

This equation can be integrated with respect to θ' to determine the pressure distribution around the cylinder.

$$\Delta p = \int_o^{\theta'} \frac{dp}{d\theta'} d\theta'$$

The following integral is required: (Ref. 7)

$$\int_o^{\theta'} \frac{(1-\alpha-e \cos \theta')}{(1-e \cos \theta')^3} d\theta' = \frac{-e \alpha \sin \theta'}{2(1-e^2)(1-e \cos \theta')^2} + \frac{e(\beta-\alpha) \sin \theta'}{2(1-e^2)^2(1-e \cos \theta')}$$

$$+ \frac{(\beta-e^2 \alpha)}{(1-e^2)^{5/2}} \tan^{-1} (\frac{\sqrt{1-e^2} \tan \frac{\theta'}{2}}{1-e})$$

where $\beta = 2 (1-\alpha-e^2)$

The periodic boundary condition on pressure, i.e., $p(\theta') = p(\theta' + 2\pi)$, requires that the coefficient of the last term must vanish. Therefore, $\beta-e^2 \alpha=0$, provides a relationship between the flow rate (Q) and rotational speed ($\dot{\theta}$).

$$\alpha = \frac{2(1-e^2)}{2 + e^2} = \frac{Q}{ab(\frac{\omega}{2} - \dot{\theta})}$$

Using this condition to eliminate α and β produces the final relationship for the pressure distribution over the cylinder (called the Harrison Equation, Ref. 8-12).

$$\Delta p = \frac{-12\, a^2(\frac{\omega}{2} - \dot{\theta}) e \sin\theta'}{b^2(2+e^2)} \; \frac{(2-e\cos\theta')}{(1-e\cos\theta')^2}$$

One additional integration will produce the desired relationship for the forces on the inner cylinder.

$$F_r = -aL \int_0^{2\pi} \Delta p \cos\theta'\, d\theta' - aL \int_0^{2\pi} \tau \sin\theta'\, d\theta' = 0$$

$$F_\theta = -aL \int_0^{2\pi} \Delta p \sin\theta'\, d\theta' - aL \int_0^{2\pi} \tau \cos\theta'\, d\theta' =$$

$$- \frac{24\pi\mu a^3 L e}{b^2(2+e^2)\sqrt{1-e^2}} \left[(\dot{\theta} - \frac{\omega}{2}) + \frac{b}{a}(1-3\sigma)(\dot{\theta} - \frac{\omega}{2}) + \frac{b}{a}\sigma\frac{\omega}{2}\right]$$

where $\sigma(e) = \frac{(2+e^2)}{6e^2}(1 - \sqrt{1-e^2})$

For $\frac{b}{a} << 1$

$$F_\theta = - \frac{24\pi\mu a^3 L e}{b^2(2+e^2)\sqrt{1-e^2}} \left[\dot{\theta} - \frac{\omega}{2}(1 - \sigma\frac{b}{a})\right]$$

The small term, $\sigma\frac{b}{a}$ in the above expression for F_θ is the only remaining contribution of the shear stress integral indicating that the pressure term dominates.

Having obtained a description of the fluid forces on non-concentric rotating cylinders, we can now return to the dynamic equations for the rotor system.

SECTION IV

DYNAMIC EQUATIONS IN POLAR FORM

Since we found it convenient to express the fluid forces in polar co-ordinates, we choose to similarly convert the dynamic equations.

Utilize the following transformation.

Let $x = r \cos \theta$; $F_r = X \cos \theta + Y \sin \theta$

$y = r \sin \theta$; $F_\theta = Y \cos \theta + X \sin \theta$

The governing dynamic equations in polar form become

$$m(\ddot{r} - r\,\theta^2) + kr = mn\omega^2 \cos(\theta - \omega t) - mg \sin \theta + F_r$$

$$m(2\,\dot{r}\,\dot{\theta} + r\,\ddot{\theta}) = -mn\omega^2 \sin(\theta - \omega t) - mg \cos \theta + F_\theta$$

These equations are non-linear and must be linearized to continue further. Consider only small perturbations in both r and $\dot{\theta}$

$r = be + \delta(t)$

$\dot{\theta} = \Omega + q(t)$

From the analysis of the fluid forces we determined the following

$F_r = 0$

$F_\theta = m\,\Omega\, be\, C_q\, (\dot{\theta} - \Omega)$

where $\Omega = \frac{\omega}{2}\left(1 - \sigma\frac{b}{a}\right) \simeq \frac{\omega}{2}$

and $C_q = \dfrac{-24\pi\mu a^3 L}{m\Omega b^3 (2+e^2)\sqrt{1-e^2}}$

Note, the effect of the shear stress term $(\sigma\frac{b}{a})$ in the expression for Ω reduces the value of Ω to slightly below 50% of ω. This result is in qualitative agreement with the experimental data (Ref. 2).

Using these relationships and neglecting higher order terms produces

the following set of linear equations.

$$\ddot{\delta} + (\frac{k}{m} - \Omega^2)\ \delta - 2\Omega beq = n\omega^2 \cos\ (\Omega-\omega)t - g \sin\ \Omega\ t + F_r/m - (\frac{k}{m} - \Omega^2)be$$

$$2\Omega\dot{\delta} + be\dot{q} - (\Omega be\ C_q)q = -\ n\omega^2 \sin\ (\Omega-\omega)t - g \cos\ \Omega\ t$$

The right hand sides of these equations contain the forcing functions and lead to the particular solution. The homogeneous solution contains the source of the self-excited oscillation and can be obtained as follows:

Let $\frac{\delta}{be} = Ae^{\lambda t}$

and $\frac{q}{\Omega} = Be^{\lambda t}$

Setting the right hand sides to zero we obtain

$$\frac{B}{A} = \frac{\lambda^2 + (\frac{k}{m} - \Omega^2)}{2\Omega^2} = \frac{-2\lambda}{\lambda - \Omega C_q}$$

This produces a cubic characteristic equation.

$$\lambda^3 - \Omega\ C_q\ \lambda^2 + (\frac{k}{m} + 3\Omega^2)\lambda - \Omega\ C_q\ (\frac{k}{m} - \Omega^2) = 0$$

Routh's discriminant states that <u>no</u> positive real root exists provided all coefficients and determinants of this equation are positive. Hence, stable motion occurs provided

1. $C_q < 0$
2. $\frac{k}{m} - \Omega^2 > 0$

SECTION V

ANALYSIS OF RESULTS

In the previous section two conditions were derived for stable motion of the journal bearing. If the two conditions are satisfied then a steady autorotation will persists at $\dot{\theta}=\Omega=0.5\ \omega$ and r=be, producing a circular orbit. This motion is descriptive of oil whirl for which experimental evidence (Ref. 2) indicates a stable autorotation at about $\dot{\theta} \simeq 0.45\ \omega$. Our predictions indicate that C_q is always negative.

$$C_q = - \frac{24\pi\mu a^3 L}{m\Omega b^3 (2+e^2)\ \sqrt{1-e}^{\,2}} < 0$$

Hence, the first condition is always satisfied. Stable rotation is a consequence of the fact that $dF_\theta/d\dot{\theta} \sim C_q$ is negative, forcing the rotation rate to return to the autorotation value ($\Omega = \frac{\omega}{2}$) if perturbations occur on either side of this value. Also equilibrium is attained at zero torque or $F_\theta = 0$ which occurs when $\dot{\theta} = \frac{\omega}{2}$. Also, experiments show that oil whirl occurs only if $\Omega < \sqrt{\frac{k}{m}}$ (Fig. 2) which verifies the second condition.

If the second condition is violated i.e., $\Omega > \sqrt{\frac{k}{m}}$, structural divergence of the shaft occurs in which the centrifugal force of the whirl becomes greater than the elastic restoring force of the shaft. The eigenvalue, λ, becomes positive indicating a departure solution. The journal deflection will become excessive and strike the bearing wall during rotation creating unusual journal position patterns. This motion at the divergence state is descriptive of oil whip.

SECTION VI

CONCLUSIONS

Self-excited oscillations in journal bearings called oil whirl and oil whip have been predicted by obtaining an analytic description of the fluid forces and inserting these into the dynamic equations for the rotor system. Oil whirl is shown to be an autorotation, Ω, of the journal centroid at a rate of one half the shaft rotational speed. This value is in good agreement with experiment. Stable rotation is a consequence of the fact that $dF_\theta/d\dot{\theta}$ is negative forcing the rotation rate to return to the autorotation value $(\frac{\omega}{2})$ if perturbations occur on either side of this value.

Oil whip is shown to occur when the frequency of autorotation exceeds the natural frequency of the shaft system, i.e. $\Omega \geq \sqrt{\frac{k}{m}}$.

At this value structural divergence occurs and the centrifugal force exceeds the elastic restoring force. Positive eigenvalues indicate departure solutions which are in agreement with experiment.

Although simplifying approximations to the Navier-Stokes equations produced satisfactory results, it is recommended that additional research be accomplished to assess the consequences of this mathematical model.

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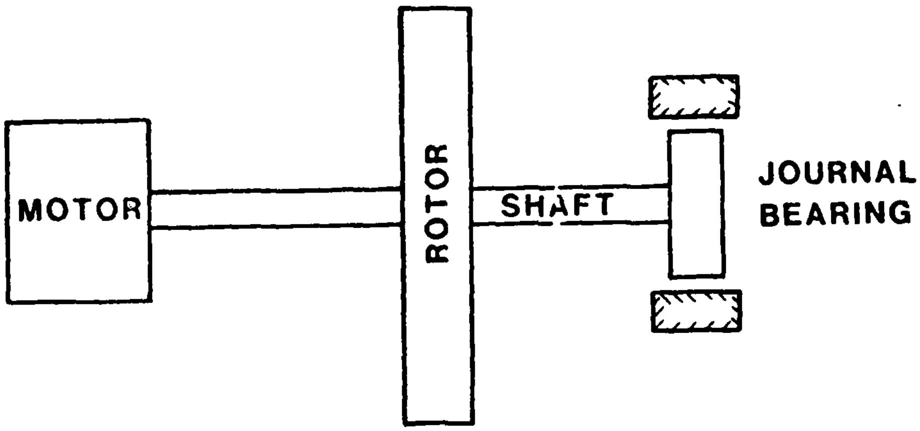

Figure 1. ROTOR SYSTEM

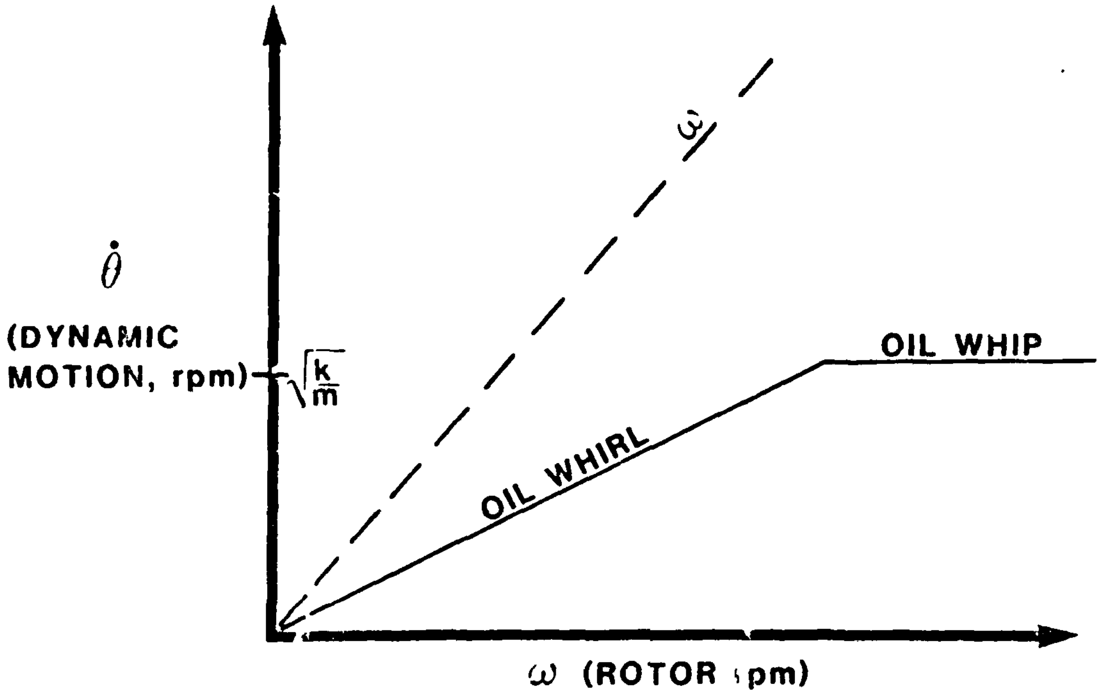

Figure 2. OIL WHIRL AND OIL WHIP

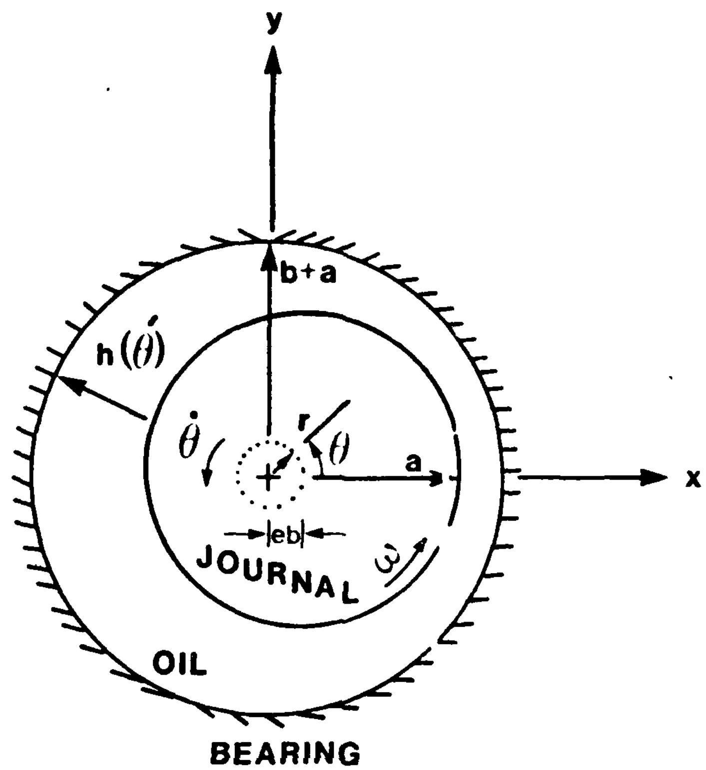

Figure 3. JOURNAL BEARING COORDINATES